Enrichments to Fiscal Commitments in the Presence of Uncertainty

by

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Abstract

Firstly, the study illustrates in a stochastic simulation model that permanent commitments to tax and spending measures can be enriched significantly without substantially raising the risk of a deficit outcome. This result is obtained in a framework where the fiscal authority is able to monitor the origin of the economic shocks and has model-consistent expectations.

Secondly, this paper evaluates whether the knowledge of the origin and persistence of the economic shocks is a key element driving this conclusion. To do this, we add to the economic uncertainty, uncertainty about the "true" persistence of the shocks and gauge whether, in that context dubbed "model uncertainty", our strategy can still enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome.

We conclude that, as time goes on, the fiscal authority may find that it has been too optimistic in enhancing its permanent commitments (*i.e.*, these additional permanent fiscal measures may increase the probability of a deficit outcome). This illustrates the difficulties of making decisions under uncertainty and the potential cost of making mistakes. This also suggests the importance of prudent behavior when committing to long-term fiscal initiatives.

Our conclusion, however, depends on the way the total uncertainty facing the fiscal authority is controlled, and on other factors such as the calibration of our model and the ability of learning of the fiscal authority. Model uncertainty may well have less effect at medium-term horizons when these considerations are introduced.

Résumé

Dans un premier temps nous montrons dans un modèle de simulation stochastique que si l'autorité budgétaire est capable d'observer l'origine des chocs économiques et a des attentes consistantes avec le modèle, des mesures budgétaires permanentes et substantielles peuvent alors être introduites sans créer une augmentation importante du risque de déficit budgétaire.

Ensuite, afin d'évaluer si ce résultat est sensible à la connaissance de l'origine et de la persistance des chocs, nous ajoutons à l'incertitude économique l'incertitude quant à la "vraie" persistance des chocs et estimons si, dans ce contexte appelé «incertitude de modèle», il est toujours possible d'enrichir les engagements permanents sans augmenter substantiellement le risque de déficit budgétaire.

Nous concluons que l'autorité budgétaire peut, dans le moyen terme, se rendre compte qu'elle a sur-réagit dans ses enrichissements d'engagements permanents, c'est-à-dire, que ces enrichissements sont accompagnés d'une hausse de la probabilité de déficit budgétaire. Ceci illustre les difficultés de la prise de décision dans un contexte d'incertitude, le coût potentiel de faire des erreurs, et l'importance d'un comportement prudent face aux engagements permanents.

Nos conclusions dépendent du contrôle du contexte d'incertitude totale dans lequel l'autorité budgétaire opère, et d'autres facteurs comme le calibrage du modèle et la capacité d'apprentissage de l'autorité budgétaire. La sur-réaction dans les enrichissements d'engagements permanents peut être réduite lorsque ces considérations sont prises en compte.

Introduction

Real GDP growth increased from an average rate of about 2.75 per cent over the period 1974-1996 to almost 4.25 per cent over the period 1997-2000. Such favorable economic shocks induced major revisions in medium-term economic forecasts of real GDP growth and in estimates of tax revenues, which substantially increased the amount of "fiscal room" (*i.e.*, fiscal surplus) available for planning purposes. Hermanutz and Matier (2000) have shown that allocating the fiscal surplus for planning purposes to enrich multi-period commitments to tax and spending measures could run the risk of a deficit outcome if the revisions turned out to be overly optimistic. This is a particular concern now in the light of more recent "negative" developments.

In a previous study [Georges and Moreau (2002)], we addressed the question of how multi-period permanent commitments to tax and spending measures could be enhanced without substantially raising the risk of a deficit outcome. We argued that this could be done to the extent that the fiscal authority took into account the persistence of the favorable economic developments. To make this rule operational we assumed that the fiscal authority was able to monitor the origin of the shocks. This, coupled with the assumption of model-consistent expectations, facilitated the task of the fiscal authority in deciding whether an observed economic shock should lead to transitory or permanent fiscal measures. This assumption is convenient for technical purposes but is clearly unrealistic.

This paper evaluates whether the knowledge of the origin and persistence of the economic shocks is a key element driving the conclusion of our previous study. To do this, we add to the economic uncertainty, "model" uncertainty about the "true" persistence of the shocks and gauge whether, in that context, our enrichment strategy can still enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome.[1]

Model uncertainty complicates fiscal planning in the following sense. Suppose that the fiscal authority observes a one-percent increase in the real output and that it correctly attributes this increase to a shock in the growth rate of potential output. If the fiscal authority mistakenly perceives this temporary growth rate shock to be permanent, then the fiscal authority will over-estimate its tax revenues and the fiscal room available for discretionary measures. We show that this mistake could lead to an overestimation of the fiscal room by about $7 billion over a five-year horizon. In our framework where decisions of enrichment to the multi-year fiscal commitment are irreversible, this can increase the probability of a deficit outcome. The objective of this paper is to evaluate

[1] In this context, the total amount of uncertainty facing the policymakers is increased. Another issue is to maintain unchanged the total amount of uncertainty and consider whether changing the *source* of uncertainty [purely additive error terms versus partly additive error terms and partly parameter uncertainty] would affect the conclusion that multi-period permanent commitments to tax and spending measures can be enhanced without substantially raising the risk of a deficit outcome. We address this issue in Subsection 2.2.

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the empirical importance of this effect and the complications that model uncertainty introduces into fiscal planning.

The structure of the paper is as follows. Section 1 sets the overall context of our study by reviewing the conditional rule, examined in our previous study, that guides the fiscal authority in the implementation of enrichments of the multi-year pre-commitment to fiscal measures. This section also explains in general terms the complication that model uncertainty brings in the framework of our analysis. Section 2 gauges the empirical importance of model uncertainty using the methodology of repeated stochastic simulations, and Section 3 concludes. An appendix describes in further details the nature of "model uncertainty" in the study -- uncertainty about the "true" persistence of output growth shocks -- and considers some calibration issues.

1. Transitory versus permanent economic and fiscal developments

In (2002) we address the question of how multi-period permanent commitments to tax and spending measures can be enhanced without substantially raising the risk of a deficit outcome. We argue that this can be done to the extent that favorable economic developments raise the scope for making discretionary program spending changes (in particular, cuts) in the future. A critical aspect of this strategy involves being able to accurately decompose transitory versus permanent components of economic developments. In Subsection 1.1, we examine more closely this decomposition through an example. Subsections 1.2 and 1.3 respectively show the fiscal implications of making an accurate versus an inaccurate decomposition.

1.1 Transitory versus permanent economic developments

The decomposition between transitory and permanent shocks can best be understood when focusing on a subset of our model – a standard trend / cycle decomposition for the economic activity. Box 1 presents the five key equations of this framework.[2]

Box 1. A standard trend / cycle decomposition of the economic activity

(1)	$y_t = y_t^p + y_t^c$	
(2)	$y_t^c = \alpha_1 y_{t-1}^c + \alpha_2 y_{t-2}^c + Z_t + \varepsilon_t^c$	$(\alpha_1 > 1, \alpha_2 < 0)$
(3)	$y_t^p = y_{t-1}^p + 0.25\mu_t + \varepsilon_t^p$	
(4)	$\mu_t = \mu_t^{tar} + (1-\lambda)(\mu_{t-1} - \mu_{t-1}^{tar}) + \varepsilon_t^\mu$	$(0 \le 1\text{-}\lambda \le 1)$
(5)	$dp_t = \beta_1 (y_t^c + y_{t-1}^c) + W$.	$(\beta_1 > 0)$

[2] The trend / cycle decomposition given by equations (1) to (4) is very similar to the one proposed by Clark (1987). The most important assumption in the model is the independence of random shocks in the potential and cyclical components. While the existence of a correlation between cyclical and secular innovations cannot be ruled out, it may be small and, as suggested by Clark, approximated by a value of zero.

Equation (1) says that the (log of) real GDP, y_t, is decomposed into a cyclical component, y_t^c (the output gap), and its potential level, y_t^p.

The cyclical component, equation (2), is modeled as an auto regressive process of order two [ARZ(2)], where Z represents all other variables (the real exchange rate, the real cost of funds, etc...) explaining the cyclical output and $\varepsilon_t^c \sim (0, \sigma_c)$ is a random error term.

The potential output [equation (3)] is assumed to evolve over time as a random walk with trend μ_t, where $\varepsilon_t^p \sim (0, \sigma_p)$ is a random error term.[3]

The deviations of the stochastic trend μ_t from its deterministic component μ_t^{tar} (or steady-state growth rate) are modeled as an AR(1) process given in (4), where $\varepsilon_t^{\mu} \sim (0, \sigma_{\mu})$ is a random error term. μ_t^{tar} is a time-varying deterministic trend (or drift) that declines gradually from a rate of 2.8 percent in 2001 to 1.5 percent in 2050. This is intended to capture the expected decline in labor market participation rates associated with the impending demographic trends.[4] The persistence of the deviations of μ from μ^{tar} is captured by the autoregressive parameter (1-λ). There are two extreme cases. The deviations are purely transitory if (1-λ) is set equal to zero, and entirely permanent if (1-λ) = 1.

Finally, equation (5) is a Phillips curve. Inflation dp_t is a function of this and previous quarter's output gap as well as other variables W (*e.g.* past and expectations of future inflation).

In this standard context, we ask the following question. What would happen if a shock raised the level of real output by one percent, at, say, time t? The answer depends critically on the source of the increase in the real output. The impact period following this shock is shown in Figure 1a where it is assumed that the economy is initially at point 1 with an output gap that is closed ($y^c = 0$) and with zero inflation ($dp = 0$). If the real output (y) increases by 1 percent it may be because either the potential output (y^p) or the cyclical output (y^c) has raised by 1 percent (equation 1). If the increase is due to an increase in the cyclical output through the shock ε^c (equation 2) the economy shifts from point 1 to point 2 along the Phillips curve. The positive output gap generates inflation.[5]

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[3] Given that the trend μ is an annualized rate but the model is quarterly, a scaling factor (0.25) is introduced.

[4] Note that in the limiting case when σ_p and $\sigma_{\mu} = 0$, y^p will follow a deterministic trend and the system (1)-(4) allocates all short run variation in y to the stationary component y^c. On the other hand, when σ_c and $\sigma_{\mu} = 0$, the system (1)-(4) allocates all short run variation in y to the level of y^p, making it similar to the model of Nelson and Plosser (1982).

[5] After the initial shock ε^c, let suppose that for all future periods $\varepsilon^c = 0$. Due to the autoregressive nature of the process on y^c (equation 2), y^c will slowly revert to its previous level (Point 1). During this dynamic, the real output and the inflation gradually comes back to their initial level. The price level, however, will permanently remain above what it was before the initial shock ε^c.

If the increase in real output originates in an increase in the potential output, the economy stays at point 1 (y and y^p increase by the same amount such that y-y^p stays unchanged). Because, in our setting, this shock has no impact on the output gap, there is no inflation.[6]

Figure 1a. Shock to the cycle versus potential output

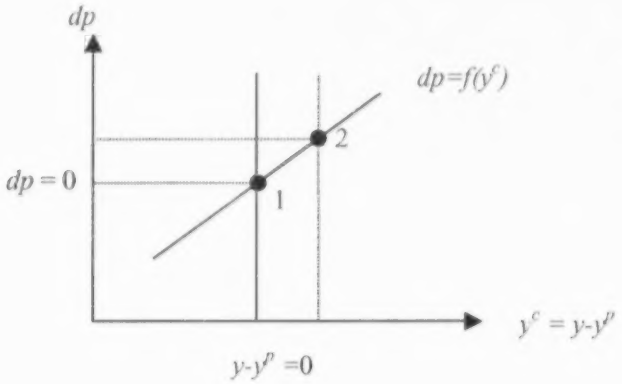
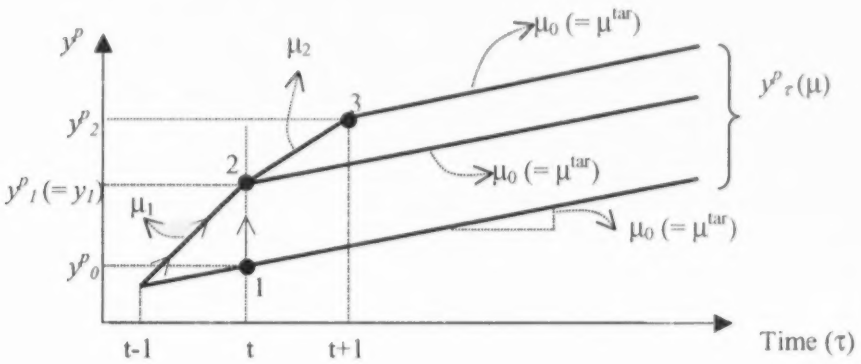


Figure 1b. Shock to the growth rate versus level of potential output



The increase in the *level* of the potential output can itself occur with or without an increase in its *trend* (or growth rate). The shock ε^p (equation 3) implies an increase in the level of potential output with no change in the growth rate, whereas the shock ε^μ (equation 4) implies an increase in the level of potential output through an increase in the growth rate μ that eventually pushes up the level of the potential output (equation 3).

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[6] After the initial shock, $\varepsilon^p > 0$, let suppose that for all future periods $\varepsilon^p = 0$. Due to the random walk assumption of the process on y^p (equation 3), y^p will permanently remain at this new higher level. Therefore, the real output also remains permanently at its new higher level. The price level remains unchanged because no inflation is generated in any period.

Figure 1b illustrates these two distinct cases. In the absence of shock, the potential output would grow over time at rate μ_0 -- the slope of the lowest path. Now suppose a shock that raised the level of real output to y_1 in time t. If the shock is due to the additive error term ε^p, the economy will be pushed to point 2. From point 2, the potential output (and with it, the total real output) continues to grow at rate μ_0 along the middle path. This is an example of an increase in the level of the potential output that originates without shock in the growth rate.

Alternatively, if the shock originates from the additive error term ε^μ, the economy will be pushed to point 2 because of the increase in the growth rate from μ_0 to μ_1 in the period t-1 to t. Numerous assumptions can be made about the persistence of the shock on the growth rate. One such assumption is illustrated by the path 2-3 and continued with the upper profile. In this case the shock is not permanent, and it gradually reverts to the initial rate μ_0 ($= \mu^{tar}$), after slowing down to μ_2 in the period t to $t+1$. [In equation (4), the persistence of the deviations of μ from μ^{tar} is captured by the parameter $(1-\lambda)$. There are two extreme cases. The deviations are purely transitory if $(1-\lambda)$ is set equal to zero, and entirely permanent if $(1-\lambda) = 1$.]

Figure 2 shows the persistence of each of these three shocks (which occurred at time t = 2001Q1) on the real output y for *future* periods. As can be seen, a shock to the cyclical output, y^c, has no permanent effect on the level of the real output. After about six quarters, the real output reaches the level it would have had in the absence of shock. On the other hand, a shock on the level of the potential output has a permanent effect on the real output. The magnitude of this effect is larger when it operates through an increase in the trend that slowly reverts to its initial level than when it operates through a constant trend. Note that in the case of the shock ε^μ, we assume that 50% of the deviation of the output growth rate from the initial (steady-state) growth rate is eliminated every period $[(1-\lambda) = 0.5]$.

Figure 3 shows the impulse response of the price level to the one percent increase in the real output in 2001. If the shock is generated through a shock in the output gap (ε^c), this will imply a permanently higher price level. On the other hand, if this shock is generated through a shock on the level (ε^p) or on the growth rate (ε^μ) of the potential output, there is no price dynamics because it is assumed that the output gap remains unchanged.

An important message that we want to convey from Figures 2 and 3 is the amount of information that the fiscal authority is assumed to have when faced with shocks. For example, after observing the increase in real output, the fiscal authority is always able to evaluate whether it originates in a shock to the level (ε^p) or growth rate (ε^μ) of potential output or to the output gap (ε^c). It is also assumed that the fiscal authority knows the persistence of the shocks, that is, the impulse responses over the planning horizon (*e.g.*, Figures 2 and 3). This is a standard assumption in stochastic simulation methodology that also underlies our previous study (2002).

This assumption makes fiscal planning easy because, by knowing the origin of the shocks and their true persistence, the fiscal authority can easily decide whether economic developments (the shocks) should lead to transitory or permanent fiscal developments. We show in Subsection 1.3 how the task of the fiscal authority is made more complex once uncertainty is introduced on the exact impulse response. Before this, Subsection 1.2 continue to build some intuition of our model by explaining how the fiscal authority reacts to economic developments in a context of full certainty about the origin and persistence of the shocks.

Figure 2. Impulse responses of the real GDP (shock minus control)

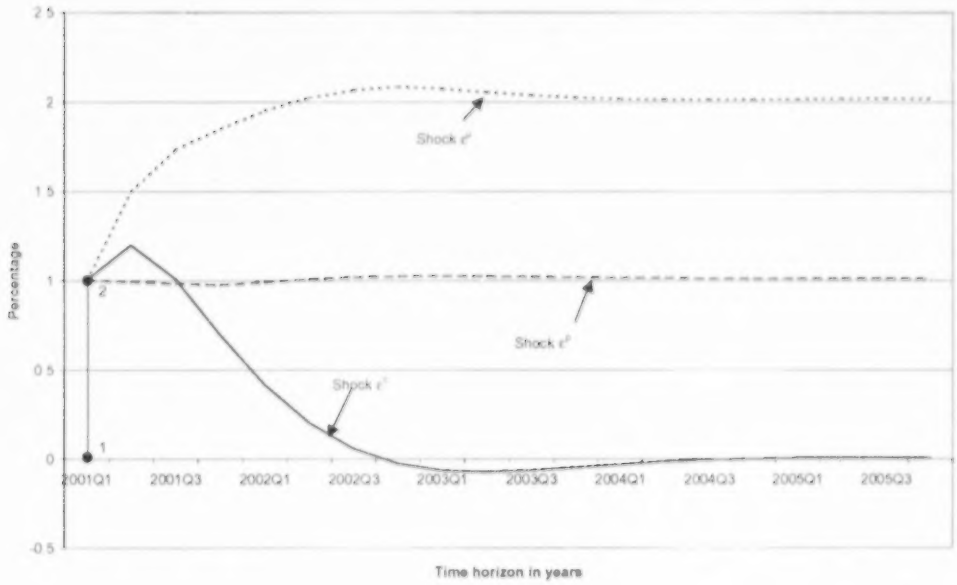
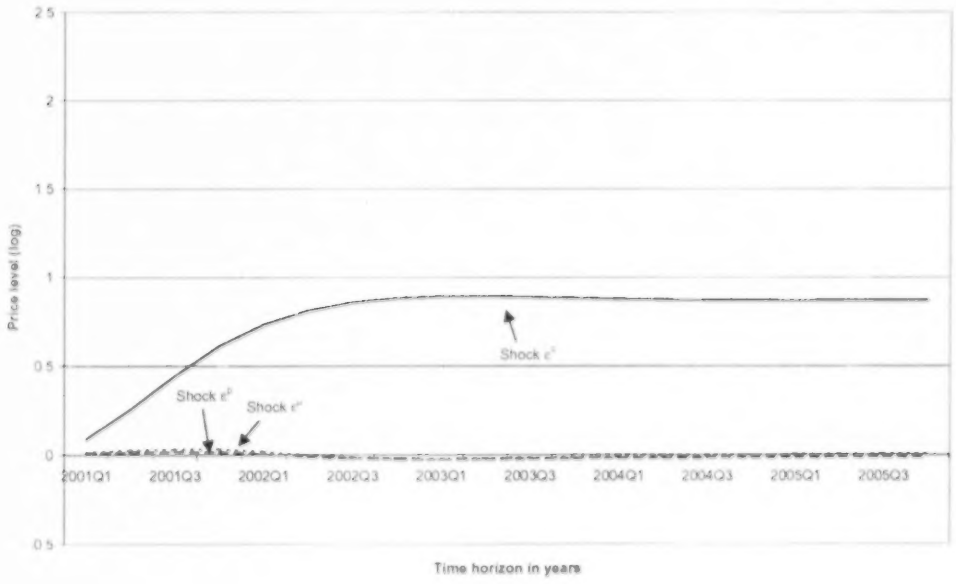


Figure 3. Impulse responses of the price level (shock minus control)



1.2 Transitory versus permanent fiscal developments

Once we assume that the fiscal authority knows the origin of the shocks and the impulse response for the real output and the price level, it automatically knows the tax implications of the shock. Indeed, in our model, we assume that the ratio of tax revenue to nominal output is a constant (about 15%). Hence, the fiscal authority is able to project the tax revenues impact of a specific shock, and to revise the projection as new shocks arrive in the economy. Figure 4 shows the tax implication of the three shocks considered above.[7] We can infer from Figures 2 and 3 that the *nominal* income ($y + p$) experiences the largest permanent increase after a shock on the growth rate of potential output (ε^{μ}) whereas the shocks on the level of potential output (ε^{p}) or on the cyclical output (ε^{c}) both imply (by coincidence) approximately the same permanent increase in nominal income.[8] Hence, the most ambitious revision in tax projection is associated with the shock (ε^{μ}) whereas the shocks (ε^{p}) and (ε^{c}) involve eventually about the same tax revenue profiles, as shown in Figure 4.

If a shock originates from the demand side (ε^{c}), creating an output gap, there are also important transition effects that imply a revision in the fiscal planning. First, the central bank attempts to close the gap by adjusting its short-term interest rate. The interest rate increase following an increase in the real output above potential has repercussion on the debt service burden. This is shown in Figure 5. The impact of a short-term interest rate increase may persist for some time because of the maturity structure of the public debt and our assumption on the term structure of interest rates.[9] Second, a counter-cyclical response originates in the automatic stabilization of the economy. In particular, an economy that is overheating entails lower spending drawn from the employment insurance program. This is shown in Figure 6 where the excess demand entails lower spending than otherwise (and hence the negative sign). Although the shocks ε^{μ} and ε^{p} do not generate an output gap, the paths in Figure 6 reflect the initial conditions that a positive output gap existed at the end of 2000 in the Canadian economy.

The budget process requires estimating the "fiscal room" or expected surplus (net of fiscal prudence) generated by the shocks ε^{μ}, ε^{p}, or ε^{c} as:[10]

$$\text{Expected surplus (net of prudence)} = TAX - PSC - PSP^{B} - DS - CR. \tag{6}$$

In other words, the fiscal authority estimates the available fiscal room by projecting the tax revenues (TAX), the amount of debt service (DS), and the amount of program spending due to the automatic stabilization linkage (PSC). PSP^{B} represents all direct program spending and transfers (unrelated to the automatic stabilization linkage) *before* the introduction of new fiscal initiatives (and hence the superscript B). The "no shock"

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[7] Figures 4 to 9 are presented in levels instead of "shock minus control" paths.

[8] The nominal income is defined here as $yn = y + p$ because y and p are respectively the logarithms of the real GDP and the price level. The nominal income is obtained by adding vertically the two corresponding profiles for y and p in Figures 2 and 3.

[9] See Hostland (2001) and Georges (2002) for a description of these channels.

[10] This is also referred to as the fiscal surplus for planning purposes.

path in Figure 7 represents PSP^B. This is what the fiscal authority had planned to spend over the fiscal horizon in absence of any shocks (*i.e*, in absence of the shocks ε^μ, ε^p or ε^c).[11]

Finally, our model assumes that the fiscal authority introduces a prudence factor or contingency reserve (CR) that introduces a wedge between expected revenues (TAX) and spending ($PSC+PSP^B+DS$), a buffer that, to a certain extent, can prevent the fiscal authority from having to post a deficit outcome in case of an unexpected negative shock during the next fiscal year.

The fiscal decision about *additional* fiscal measures (program spending or tax cuts) unrelated to the automatic stabilization linkage may then be viewed as a discretionary (decision) variable that, in our model, is supposed to exhaust the fiscal room, that is:

(7) $\Delta PSP=PSP^A-PSP^B$ = Exp. surplus (net of prudence) = $TAX- PSC - PSP^B - DS - CR$,

where PSP^A is the amount of direct program spending and transfers (unrelated to automatic stabilization) including all previous measures (PSP^B) plus the fiscal initiatives ΔPSP announced in this new budget.[12] The superscript A refers to the level of program spending *after* the inclusion of new measures. Negative economic shocks can lead to negative initiatives (*e.g.*, cuts in program spending such that $PSP^A<PSP^B$).

The paths for PSP^A implied by the three shocks (ε^c, ε^p, ε^μ) under consideration are given in Figure 7 by the three upper paths. The distance or area between each of these paths and the "no shock" path (or PSP^B) represents the additional fiscal measures (ΔPSP) announced in the new budget conditional on the shock under consideration. Given our assumption that additional fiscal measures are introduced to exhaust the fiscal room, this area is also, by construction, equal to the fiscal room (net of prudence) generated by each of the three shocks. The shock on the growth rate of potential output (ε^μ) leads to the largest fiscal room and thus to the largest increase in discretionary spending (or tax cuts), whereas the shocks on the output gap (ε^c) and the level of potential output (ε^p) eventually lead to about the same fiscal room and thus to similar increases in program spending.

The expected surplus given in (6) must be reduced by the value of ΔPSP to obtain the fiscal authority's expected surplus (or budget balance) net of prudence *after* the introduction of fiscal measures:

(8) $\qquad BAL\ (net\ of\ prudence) = TAX - PSC - PSP^B - \Delta PSP - DS - CR,$

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[11] This way to measure the expected surplus is similar to the budgetary surplus *before measure* as computed, say, in Table 7.2, p.142 of the Budget Plan 2001. The only difference is that in equation (6) we subtract the prudence factor to obtain an expected surplus *net of prudence*.

[12] Our measure, ΔPSP, is equivalent to the spending and revenue initiatives identified, say, in Table 7.3, p.144 of the Budget Plan 2001.

which is equal to zero when we substitute the value of ΔPSP given in (7) into (8).[13] Hence, in our model, the fiscal authority targets a budget balance after the introduction of fiscal initiatives equal to the prudence factor (or a budget balance net of prudence equal to 0).

[13] The budgetary balance including budget measures as given in equation (8) is equivalent to, say, the budgetary balance given in Table 7.4, p.147 of the Budget Plan 2001.

Figure 4. Tax revenue projections under alternative shocks

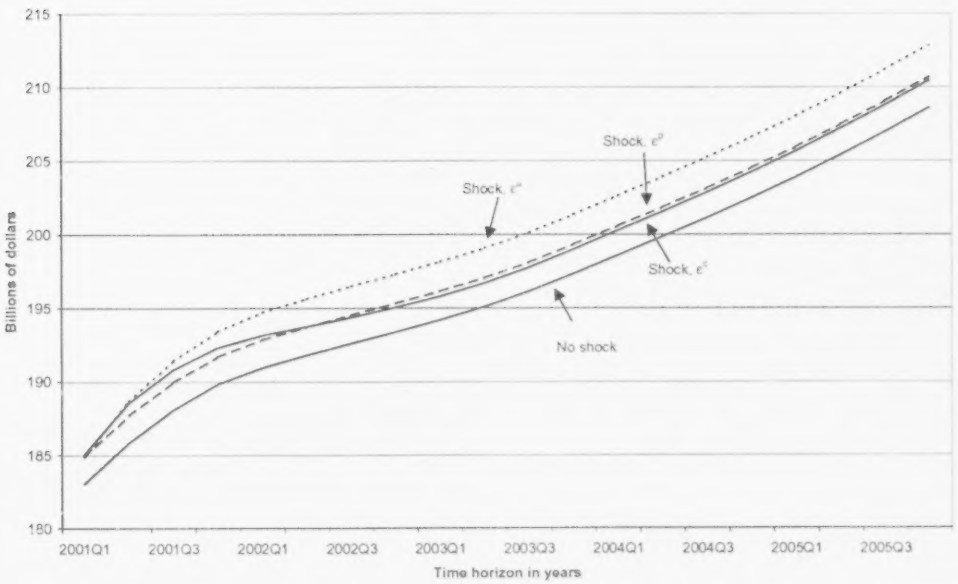


Figure 5. Debt service projections

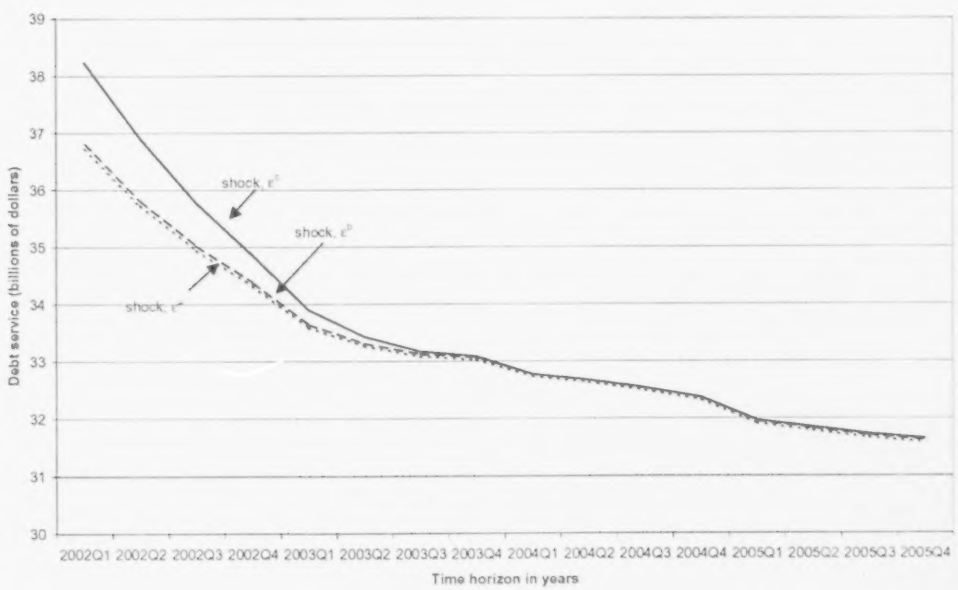


Figure 6. Automatic stabilisation spending (PSC)

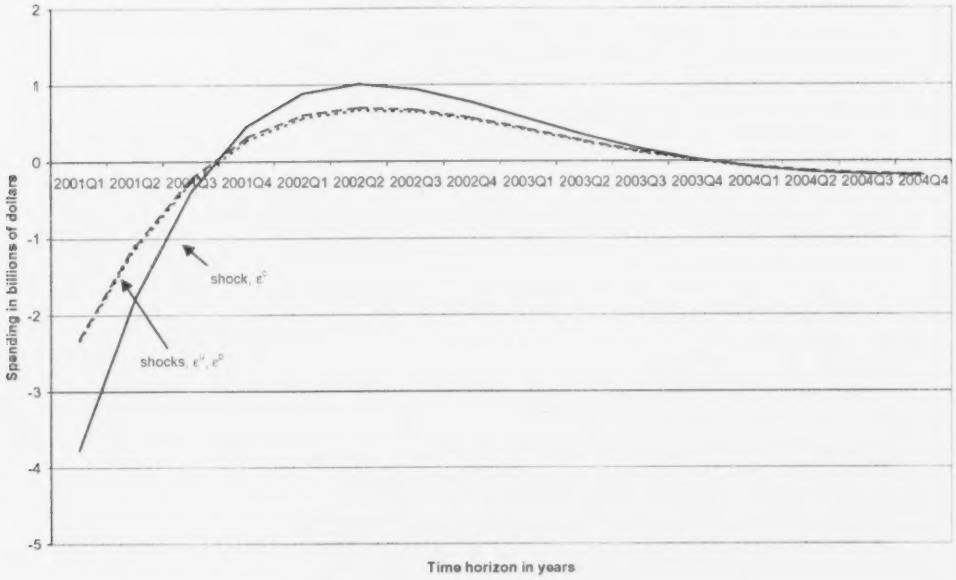
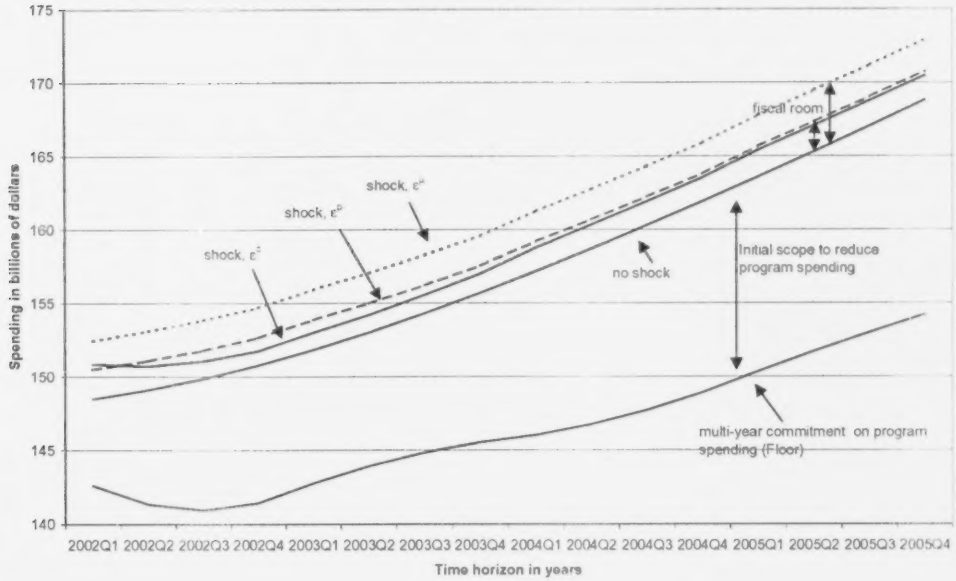


Figure 7. Discretionary program spending (PSP)



Reversible versus irreversible spending commitments

The upper paths in Figure 7 correspond to projections of total program spending (PSP^A) conditional on the observed shocks, but assuming no further shocks in the future. This raises the issue of reversibility (or irreversibility) of program spending in case of *future* negative shocks. The amount of "discretionary" program spending, PSP^A, may be viewed as a transitory or reversible item in the sense that there is no commitment for that level in *future* years of the planning horizon.[14] In particular, if economic growth turned out to be weaker than anticipated during the next fiscal year, then, this would entail a downward revaluation of the fiscal room available and thus a downward shift of the profiles for PSP^A in Figure 7.

In our model, the fiscal authority has a multi-year "floor" or explicit pre-commitment on the minimum value of program spending (illustrated in Figure 7 by the lowest path). The fiscal authority cannot respond to large negative shocks by reducing spending below this floor. The multi-year pre-commitment can therefore be viewed as the amount of permanent or irreversible spending in the economy. Introducing a floor implies that it is not the *total* amount of program spending (PSP^A) that should be viewed as transitory or reversible, but the amount of program spending above the floor (PSP^A – Floor). This magnitude, in our model, is called the scope to reduce program spending and is illustrated in Figure 7 by the distance between the relevant path for PSP^A and the floor. The distance between the "no shock" path (PSP^B) and the floor gives the "initial" scope, that is, the scope before the arrival of the economic shocks.

A final feature of our model is the ability of the fiscal authority to enrich its multi-year commitment. In other words, an upward revision of the floor is allowed if the fiscal authority considers that circumstances justify this enrichment. On the other hand, by definition, a downward revision of the floor is never allowed. Hence, in our model, a rule allows the fiscal authority to determine each year whether or not there will be an enrichment in the floor and, if there is, the rule determines the magnitude of the shift. According to the rule, multi-year pre-commitments will be enriched to the extent that favorable economic developments raise the magnitude of the scope for making discretionary changes in program spending in the future.

For example, the shock ε^{μ} leads to a fiscal room that allows a higher amount of program spending than otherwise (the "no shock" path), and given by the upper path in Figure 7. With an unchanged floor this necessarily increases, by that same amount, the magnitude of the scope for making discretionary changes in program spending.[15] This suggests that the fiscal authority can enrich the floor by that amount and maintain the scope to its initial level. Figure 8 illustrates the enrichment of the floor implied by the three shocks ε^{c}, ε^{p}, and ε^{μ}. The enrichment critically depends on the source of the increase in the real output, that is, whether it originates in a shock to the cyclical output, to the potential level of the output with constant trend, or to the growth rate of potential output.

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[14] The fiscal authority is, however, committed to the level of PSP^A for the current fiscal year of the budget.
[15] The scope effectively increases from PSP^B - floor to PSP^A - floor.

Figure 8. Enrichment of multi-year fiscal commitment

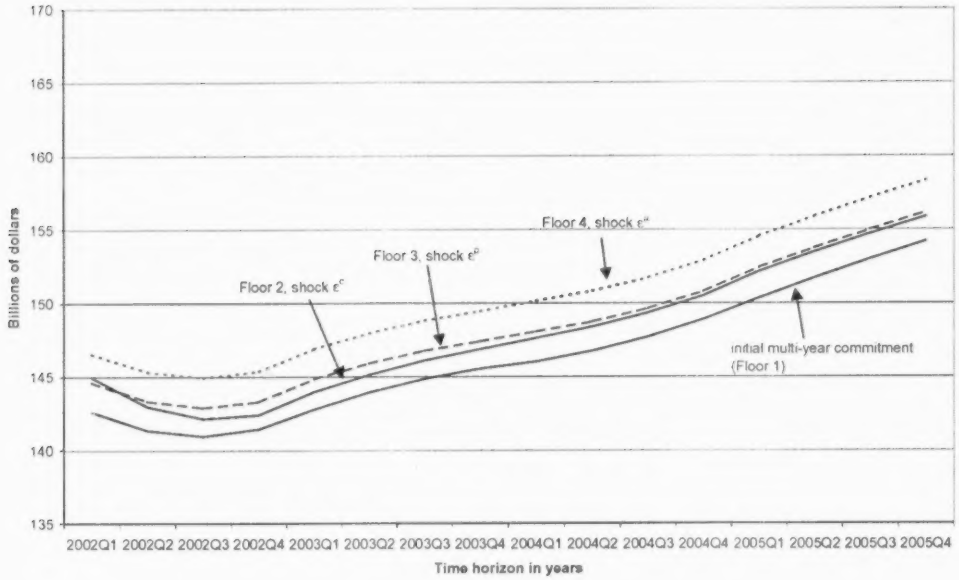
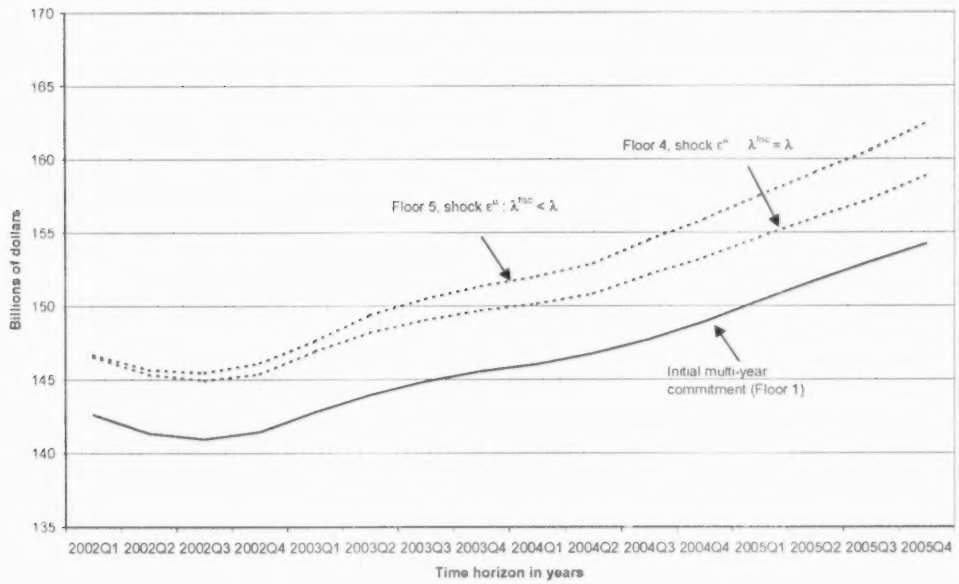


Figure 9. Enrichment with and without model uncertainty



Although the projected path for PSP^A (in Figure 7) can be shifted down if future negative economic shocks cause the fiscal authority to revaluate downward the estimated fiscal room, the enrichment in the floor (in Figure 8) is permanent. Unexpected negative shocks can therefore generate a reduction of the scope, which increases the probability of a deficit outcome. Our previous study (2002) gauges that effect using a stochastic simulation methodology, and concludes that the enrichment strategy can enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome. A caveat to this conclusion is that the fiscal authority is assumed to know the source of the shocks and their true persistence. In reality, it is unlikely that the fiscal authority would be able to disentangle the origin of the shocks and to gauge their persistence with much accuracy.

In this new study, we want to evaluate whether the knowledge of the origin and persistence of the economic shocks is a key element driving the conclusion of our previous study. To do this, we introduce uncertainty about the "true" persistence of the shocks and gauge whether, in that context dubbed model uncertainty, our enrichment strategy can still enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome. Before turning to simulations results, we show in the next subsection how complications entailed in introducing model uncertainty affect the fiscal planning.

1.3 Model uncertainty and fiscal planning

Figure 8 illustrates the impact of introducing model uncertainty on the decision to enrich the multi-year fiscal pre-commitment. Suppose that an economic shock is mistakenly interpreted by the fiscal authority as originating from an increase in the growth rate of potential output (ε^{μ}) instead of an increase in the level of potential output (ε^{p}). This could lead to an enrichment suggested by Floor 4 instead of Floor 3. The enrichment would therefore be more ambitious than it should have been.

The forecast error on the source of the shock would lead to permanent effects in our model. In the example of Figure 8, the upgrade to Floor 4 is permanent or irreversible because it becomes the new pre-committed minimum level of program spending, with no possibility to backtrack on it. However, as the fiscal authority realizes its forecast error (as it collects lower tax revenues than anticipated), it will reevaluate downward the available fiscal room and the discretionary fiscal measures (PSP^A). This downward revision coupled with the irreversible upgrade in the floor effectively reduces the scope for discretionary measures. This will result in an increase in the probability of a deficit outcome.

There is a variety of ways to introduce model uncertainty. As just shown, the fiscal authority could confuse the source of the shock, for example, by confusing a shock on the level for a shock on the growth rate of the potential output. An alternative is to assume that the fiscal authority knows the source of the shock, but confuses its "true" persistence and thus makes errors when evaluating the path of the impulse response. For example, in Figure 2, we assume that after a shock ε^{μ}, the potential output (and thus the real output)

eventually reverts to the steady-state rate at a given speed – 50% of the deviation is corrected every period. In terms of equation (4), this means that $\lambda = 0.5$. Model uncertainty can be introduced, in this particular case, by assuming that the fiscal authority makes errors on the "true" reversion speed.

To illustrate, suppose again that the fiscal authority observes a one-percent increase in the real output in 2001 and correctly attributes it to a shock to the growth rate of the potential output. Suppose also that the fiscal authority mistakes the shock in the growth rate for being a permanent one by setting the reversion speed to zero ($\lambda^{fisc} = 0$ instead of the "true" parameter λ=0.5). Figure 9 illustrates the implication of this mistake on the enrichment of the commitment to a minimum level of fiscal measures. The floor shifts upward from Floor 1 to Floor 5 instead of Floor 4, which would be the appropriate response if the fiscal authority knew the true reversion speed. The area between Floors 4 and 5 is about $7 billion over a five-year period. This shows that the fiscal authority overestimated the fiscal room by $7 billion, which translated into an over-ambitious enrichment in its multi-year commitment of $7 billion. This will eventually turn into a reduction in the scope for discretionary measures of $7 billion as the fiscal authority observes at the end of the year that its tax revenues were overestimated.

Independently of the source of model uncertainty, the main point that we want to make is that model uncertainty may have consequences for the amount of permanent fiscal measures introduced and for the probability of a deficit outcome. Section 2 gauges this impact using repeated stochastic simulations.

The simulation experiment is intended to show how fiscal planning is complicated by uncertainty about the model (parameter) when it is added to the amount of uncertainty captured by the additive error terms of our stochastic simulation model. In other words, the uncertainty facing the policy makers is increased by the amount of model uncertainty.

Another question is to investigate whether the enrichment strategy is sensitive to the source of uncertainty. In Subsection 2.2, we introduce model uncertainty in a way that does not raise the total amount of uncertainty facing the policy-makers and consider whether this affects the conclusion of our previous study that multi-period permanent commitments to tax and spending measures can be enhanced without substantially raising the risk of a deficit outcome.

2. Simulation results

2.1 Model certainty versus model uncertainty

In this section, we want to evaluate whether knowledge of the origin and persistence of economic shocks is a key element driving the conclusion that enhancing the fiscal authority's pre-commitment to permanent measures is possible without substantially increasing the risk of a deficit outcome.

To do this, we introduce model uncertainty about the "true" persistence of the shocks and gauge whether, in that context, our enrichment strategy can still enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome.

In particular, we examine the consequence for the fiscal authority of overestimating the true persistence impact of output growth shocks (ε^{μ}). To do this, we analyze the two scenarios given in Table 1. In both scenarios the value of the "true" persistence parameter λ (as given in equation 4) is assumed to be equal to 0.5. This means that 50% of the deviations of the trend (μ) from its deterministic component (μ^{tar}) will persist from one period to another. In Scenario 1, we assume that the fiscal authority knows the true persistence parameter such that $\lambda^{fisc} = \lambda = 0.5$.

In Scenario 2, the fiscal authority makes errors by overestimating the persistence of shocks, such that $\lambda^{fisc} = 0.01 < \lambda = 0.5$. In this case, the fiscal authority considers that the deviations from the deterministic trend follow all but a random walk. In other words, the fiscal authority's conditional expectation of the persistence of the shock is larger than the model-consistent expectation. In essence, the fiscal authority is too "optimistic" in its economic and fiscal projections following an unexpected high growth rate of the economy and too "pessimistic" following a period of slower growth than expected. (Appendix A1 explains in further details this assumption.)

Table 1. Model certainty versus model uncertainty

λ^{fisc} \ λ		$\lambda = 0.5$
Scenario 1	$\lambda^{fisc} = 0.5$	Model certainty
Scenario 2	$\lambda^{fisc} = 0.01$	Model uncertainty

These forecast errors will have repercussions for the fiscal plan. Periods of higher growth than expected will be followed by a budget plan that overestimates the projections for tax revenues (TAX) (and underestimates the projections for the automatic stabilization spending (PSC)). This will, for the same fiscal prudence (CR), lead to (i) an overestimated "fiscal room", (ii) the implementation of excessive additional fiscal measures (ΔPSP) and (iii) an over-ambitious enrichment in the irreversible multi-year fiscal commitment. This may have consequences on the probability of a deficit outcome over the planning horizon. The objective of this section is to gauge the empirical importance of this effect.

To this end, we use a stochastic simulation model (SSM) that captures the uncertainty inherent in the economy. Repeated (500) simulations of the same planning horizon generates a large number of realizations that permit the construction of confidence intervals, or a range of possible outcomes with corresponding probabilities. For example, by providing a probability distribution of the budget balance, this methodology permits the calculation of the risk, or probability, of having a deficit in any year of the planning horizon. Hostland (2001) describes in depth the stochastic simulation model of the Canadian economy that we have used for our study.

We first simulate our model of the Canadian economy (in a context of model certainty) in the absence of any strategy of enrichment of the multi-year fiscal commitment. Hence the floor is maintained at the level given by Floor 1 in Figure 10, over the fiscal horizon. The probability of deficit outcome is given by the lowest profile in Figure 11.

We then simulate the same model but with the addition of our strategy of enrichment of the multi-year fiscal commitment (Scenario 1, Table 1). According to our simulations, the fiscal authority would, on average, add about $25 billion of (cumulative) permanent fiscal measures over a five-year planning horizon (2001 to 2006), and $83 billion over a 10-year horizon (2001-2011). This corresponds, in Figure 10, to the area [for the relevant 5- or 10-year horizons)] between Floor 1 and the profile named "updated floor, model certainty".[16] This enrichment strategy generates a probability of a deficit given by the middle line in Figure 11. Comparing the two lowest profiles in Figures 10 and 11 leads us to conclude that a significant enhancing of the fiscal authority's commitment to permanent measures is possible without substantially increasing the risk of a deficit outcome.

Figure 11bis shows the extent to which the average amount of debt reduction over a ten-year horizon varies under different scenarios. A total of $40 billion in debt repayment is attained on average in simulations conducted without any multi-year fiscal commitment to program spending and tax. This reflects a fiscal prudence set at $4 billion each year of the fiscal horizon. The average debt reduction falls to $35 billion when a multi-year commitment to permanent measures is introduced (path "without enrichment of the floor" in Figure 11bis). This is so because situations arise that prevent the fiscal authority to set aside $4 billion of prudence in those years where the multi-year commitment becomes binding. Ex post, the average prudence over a 10-year horizon is thus $3.5 billion. Finally, in the scenario where the fiscal authority enhances the multi-year commitment, the average debt reduction falls to $31 billion, which represents an average level of fiscal

[16] Floor 1 in Figure 10 is the initial multi-year pre-commitment for program spending at the start of the planning horizon (2001Q1). As the planning horizon unfolds, positive shocks may arrive, inducing the fiscal authority to enrich the multi-year pre-commitment. This generates an update of the floor. The planning horizon lasts for 10 fiscal years. Therefore, there are 10 budgets and 10 decisions to enrich or not the previous-year multi-year pre-commitment. This is considered as "one" complete simulation (or realization). After one simulation we obtain a locus representing the envelope of the updated multi-year pre-commitment By running 500 simulations, we eventually generate a probability distribution of these loci. The average updated floor in Figure 10 is the mean of this distribution. Because of the asymmetry of our rule the median differs from the mean. Based on the median, the fiscal authority would add, in average $17 ($63) billion of permanent measures over a 5- (10-) year horizon.

prudence of $3.1 billion (path "with enrichment of the floor, model certainty" in Figure 11bis). Comparing the average debt path without and with enrichments of the floor suggests an increase in debt by $0.4 billion in 2006 and by $4 billion at the end of the 10-year horizon. This shows that the fiscal authority can substantially enrich its permanent commitments without substantially changing the average debt reduction path.

Figures 10, 11 and 11bis also illustrate the impact of adding model uncertainty. In this context (scenario 2, Table 1), the fiscal authority becomes overly ambitious when enriching its pre-commitment to permanent fiscal measures, as is shown in Figure 10. For example, in a context of model uncertainty, the amount of cumulative permanent fiscal measures rises to $31 billion (instead of $25 billion) over a five-year horizon and to $116 billion over a 10-year horizon (instead of $83 billion).[17] The impact on the probability of a deficit outcome is illustrated in Figure 11, where, the probability is about 5% higher than otherwise in 2006, and 14% higher than otherwise in 2011. The impact on the average debt reduction path is shown in Figure 11bis. The average debt retirement now falls to $20 billion, which represents an average fiscal prudence of $2 billion per year. This represents a substantial change in the debt reduction plan.

We conclude that, as time goes on, the fiscal authority may find that it has been too optimistic in enhancing its multi-year commitments. This illustrates the difficulties of making decisions under uncertainty and the potential cost of making mistakes. This also suggests the importance of prudent behavior when committing to long-term fiscal initiatives.

[17] Based on median instead of mean simulation results, the amount of cumulative permanent fiscal measures raises to $22 billion (instead of $17 billion) over a five-year horizon and to $106 billion over a 10-year horizon (instead of $63 billion).

Figure 10. Average amount of enrichment: Model certainty versus model uncertainty

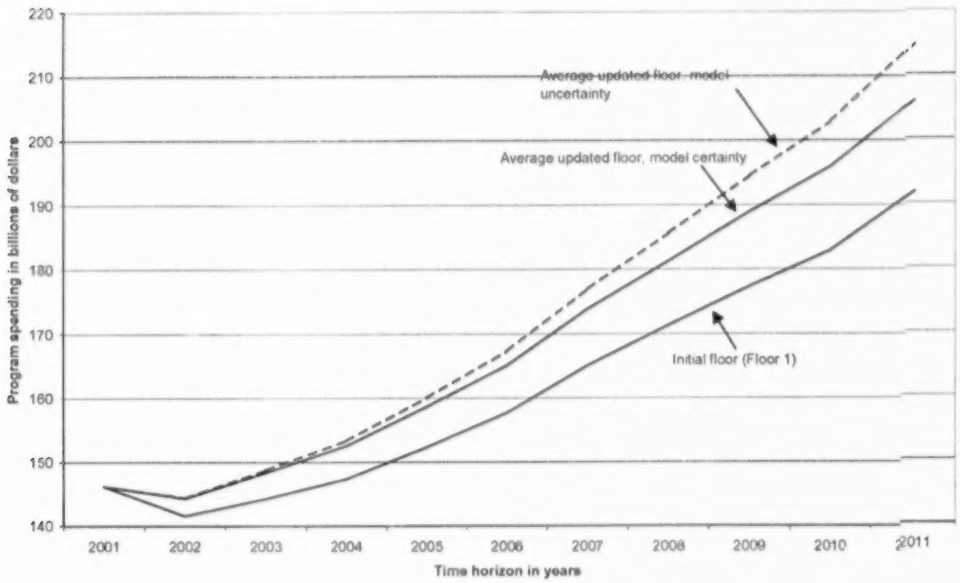


Figure 11. Probability of a deficit: Model certainty versus model uncertainty

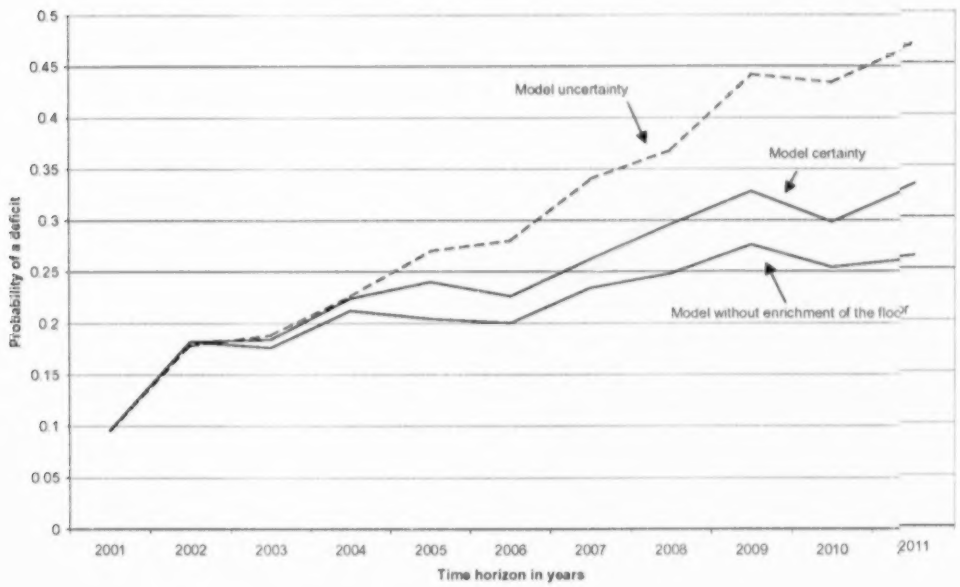
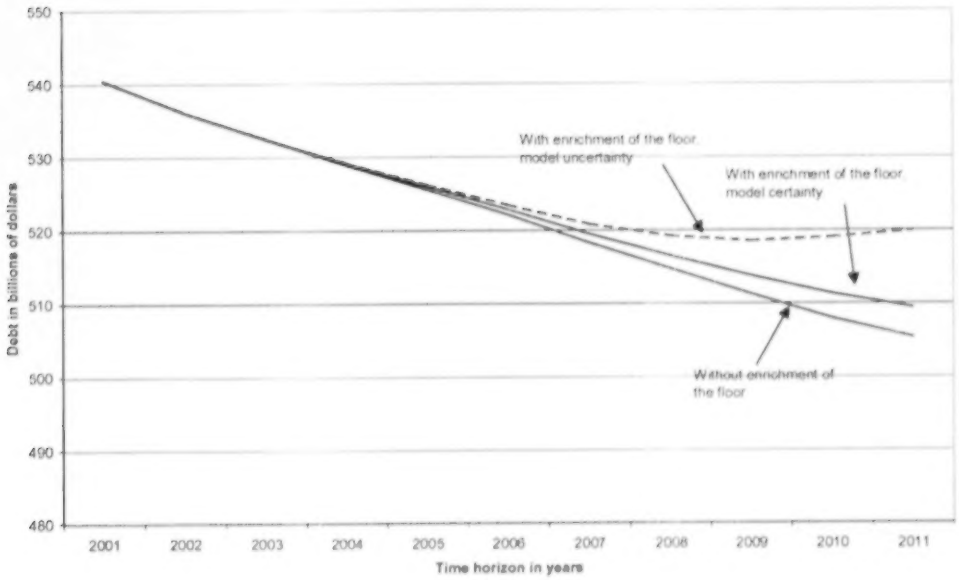


Figure 11bis. Average debt reduction path



2.2 Controlling for the total amount of uncertainty

The uncertainty in Scenario 1 (as described in Subsection 2.1) is generated through additive error terms only [*e.g.* the variance of the shocks ε^c, ε^p and ε^μ in equations (2)-(4).] whereas Scenario 2 assumes that the uncertainty about the parameter λ (model uncertainty) is *added* to the uncertainty generated by the additive error terms in Scenario 1. (See Table 2, Column 2.) In other words, Scenario 2 increases the amount of uncertainty facing the policy makers by the amount of model/parameter uncertainty.

Table 2. Sources of uncertainty versus total amount of uncertainty

	Sources of uncertainty	
	Subsection 2.1	Subsection 2.2
Scenario 1	Additive error terms: $Var(\varepsilon^\mu) = 0.75^2$	Additive error terms: $Var(\varepsilon^\mu) = 0.75^2$
Scenario 2	Additive error terms: $Var(\varepsilon^\mu) = 0.75^2$ + Parameter (model) uncertainty ($\lambda^{fisc} \neq \lambda$)	Additive error terms: $\mathbf{Var(\varepsilon^\mu) = 0.57^2}$ + Parameter (model) uncertainty ($\lambda^{fisc} \neq \lambda$)
Total amount of uncertainty	Higher in Scenario 2 than in Scenario 1	Same in Scenarios 1 and 2

This raises the following issue. We saw in Subsection 2.1 that, as time goes on, the fiscal authority may find that it has been too optimistic in enhancing its multi-year commitments, with repercussions for the probability of a deficit outcome. But this may result from the increase in the total amount of uncertainty facing the fiscal authority. We investigate this by introducing model (parameter) uncertainty in a way that does not raise the total amount of uncertainty. This requires reducing the variance of the additive error terms in Scenario 2 (Column 3 in Table 2). Hence, the "total amount" of uncertainty (additive error terms + model uncertainty) facing the policymakers in Scenario 2 remains unchanged when compared to Scenario 1 (additive error terms only).[18]

We assume that maintaining unchanged the total amount of uncertainty means that the forecasting performance of the fiscal authority should remain identical across the two

[18] We can think of the (reduced) variance of the additive error terms in Scenario 2 as capturing the amount of uncertainty due to unanticipated economic developments whereas the higher variance of the additive error terms in Scenario 1 can be thought of as capturing both the uncertainty due to unanticipated economic development and the parameter uncertainty.

scenarios. This entails reducing the variance of the additive error terms (the variance of ε^{μ}) in Scenario 2 such that the root-mean-square errors of the growth rate of the real output are equalized across scenarios.[19] There are alternative ways to think about "neutralizing" the total amount of uncertainty facing policymakers. We leave this discussion to Box 2.

Once we control for the total amount of uncertainty facing the policymakers, we find (Figures 12, 13, and 13bis), that the fiscal authority's enrichment strategy continues to be over-ambitious. As well, we continue to observe an increase in the probability of a deficit outcome over the long-term and a smaller average debt reduction, although these effects are smaller than they were before controlling for the amount of uncertainty.[20]

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[19] The root mean square error is given by: RMSE $= \sqrt{\frac{1}{T}\sum_{t=1}^{T}(gy_t^s - gy_t^a)^2}$, where gy^s-gy^a is the deviation of the fiscal authority's forecasted value for the growth rate of real output from its actual (realized/simulated) value, and T is the number of periods in the simulation.

[20] This also implies that the additive error terms in our stochastic simulation model cannot be interpreted to capture adequately parameter (or model) uncertainty and other sources of uncertainty such as unanticipated economic developments.

Box 2. Controlling for the total amount of uncertainty

Maintaining unchanged the forecasting power of the fiscal authority is one among other ways to control for the total amount of uncertainty. Another possibility is to equalize the fiscal authority's subjective probability distribution of the growth rate of the potential output (μ in equation 4) in Scenario 2 to the objective probability distribution in Scenario 1.

As shown in Appendix A3, $Var(\mu) = Var(\varepsilon^{\mu})/[1-(1-\lambda^2)]$. Thus, for the fiscal authority's subjective variance of μ in Scenario 2 to be equal to the true variance in Scenario 1(0.866^2), and given that the persistence of the shock perceived by the fiscal authority is smaller than the true value ($\lambda^{fisc} = 0.01 < \lambda = 0.5$), we need to reduce $Var(\varepsilon^{\mu})$ to 0.122^2 (from 0.75^2).

As compared to the case where we maintain unchanged the forecasting power of the fiscal authority ($\varepsilon^{\mu} = 0.57$), this is a much-reduced variance for ε^{μ} (0.122^2). This is so because this implies controlling for the amount of uncertainty *ex ante*, and therefore this does not take into consideration that the fiscal authority effectively observes at the end of each year that its forecast overshoots the actual growth rate of output. This method for controlling total uncertainty may actually overestimate the impact of model uncertainty.

Using this method to "control" for the total amount of uncertainty facing the policymakers, we find that Scenario 2 entails an average amount of enrichment and a probability of a deficit outcome given by the middle paths in Figures 12 and 13 (*i.e.*, same as in Scenario 1). We conclude in this case that model uncertainty does not challenge the proposed fiscal authority's enrichment strategy, even in the medium term (after five years).[1]

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[1] We can think of the reduced variance of the additive error terms in Scenario 2 as capturing the amount of uncertainty due to unanticipated economic developments whereas the higher variance of the additive error terms in Scenario 1 can be thought of as capturing both the uncertainty due to unanticipated economic developments and parameter uncertainty. Hence, unlike our results in the main text, the results in this box lead to conclude that introducing stochastic parameters into the simulation framework has much the same effect as increasing the magnitude of the additive error terms in the model. This result is similar to the one generated by Dupuis and Hostland (2001) who use the same stochastic framework as we do.

Figure 12. Average amount of enrichment: Additive error terms versus additive error terms and model uncertainty – Same RMSE

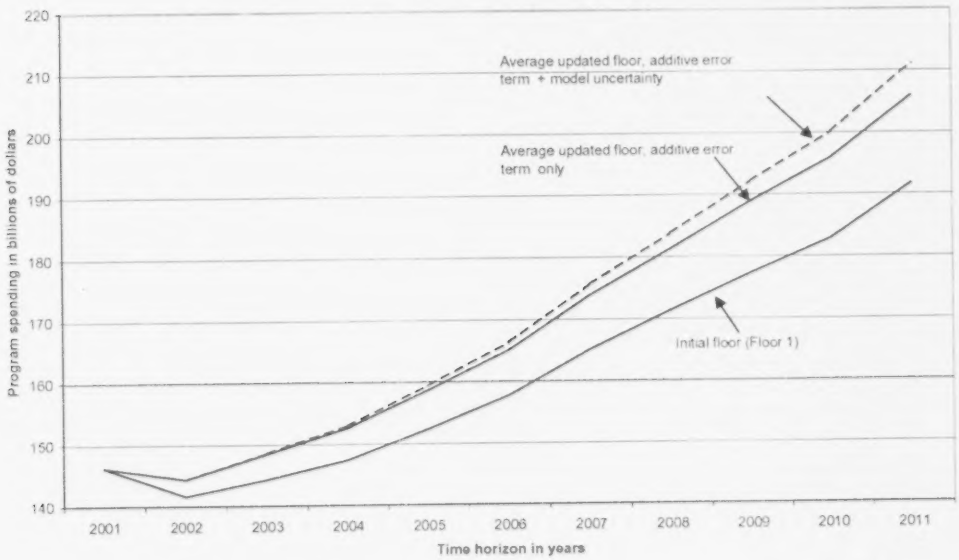


Figure 13. Probability of a deficit: Additive error terms versus additive error terms and model uncertainty -- Same RMSE

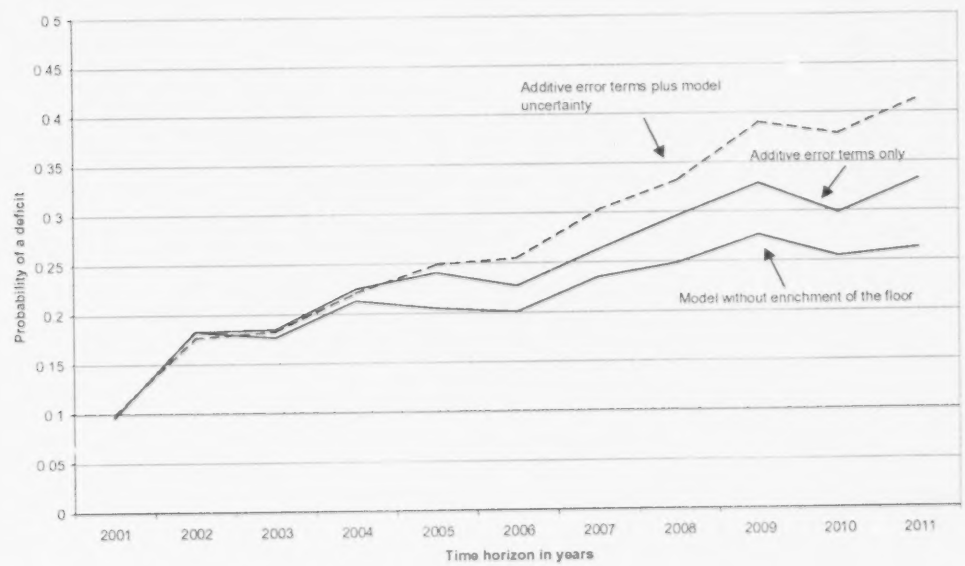
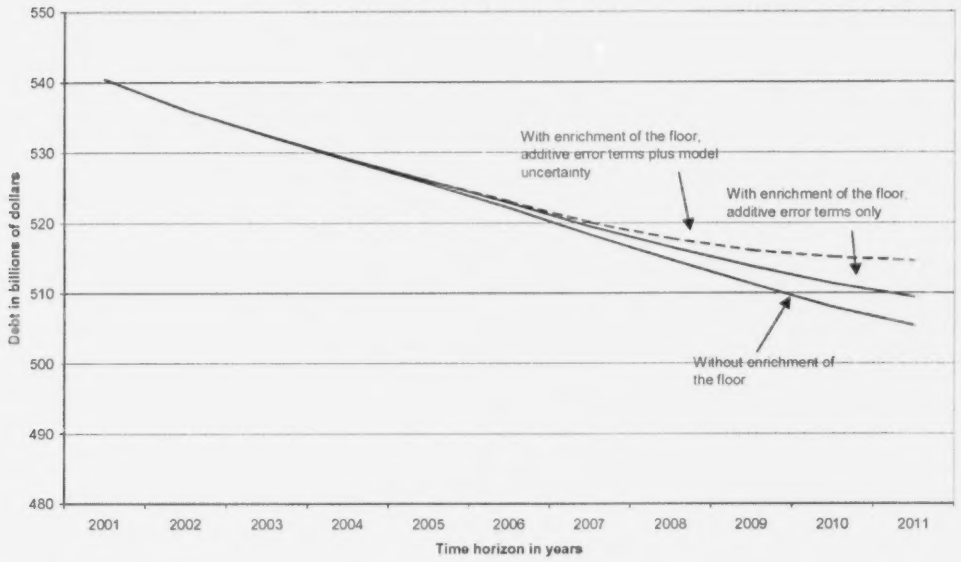


Figure 13bis. Average debt reduction path: Additive error terms versus additive error terms and model uncertainty --Same RMSE



2.3 Extensions

Learning

An assumption made in Subsection 2.1 is that the fiscal authority never learns from its mistakes. An easy way to mimic (instead of modeling) learning about the true parameter is to let λ^{fisc} tend eventually to its true value, λ, at a given speed.[21] This gives a solution (not reported) in Figures 10 and 11 bound between the results of model certainty and model uncertainty. A faster speed of learning leads to a solution closer to the results obtained under model certainty. Hence, learning naturally mitigates the impact of model uncertainty. An issue that warrants further research is whether the speed coefficient of learning could be linked to some performance, say, the forecast errors of tax revenues.

Calibration

The calibration of our model (as shown in the Appendix) implies that a 90% confidence interval surrounding the growth rate of potential output is ± 1.42%. Given our assumption of an expected value of 2.8%, this implies a confidence interval on μ given by [1.38%; 4.22%].[22] Given that this confidence interval may be viewed as slightly on the "high side", we recalibrated our model such that the 90% confidence interval for μ would be given by [2.23%; 3.36%].[23] With this new calibration, Figures 14 and 15 illustrate that adding model uncertainty has now virtually no impact, even in relatively long term horizons (10 years).

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[21] We can postulate for example that $\lambda_t^{fisc} - \lambda_{t-1}^{fisc} = \alpha(\lambda - \lambda_{t-1}^{fisc})$, where α is the learning speed coefficient. Setting $\alpha = 1$ (0) involves model certainty (model uncertainty). Values for α between 0 and 1 mimic the possibility of learning.

[22] This comes from the fact that $Prob[(\mu - 2.8)/sd_\mu > (4.22 - 2.8)/sd_\mu] = 0.05$ when $sd_\mu = 0.866$ as it is assumed in the Appendix. This confidence interval for μ is consistent with the historical period 1961Q1-1999Q4.

[23] This confidence interval for μ is consistent with the historical period 1989Q1-1999Q4.

Figure 14. Average amount of enrichment: New calibration

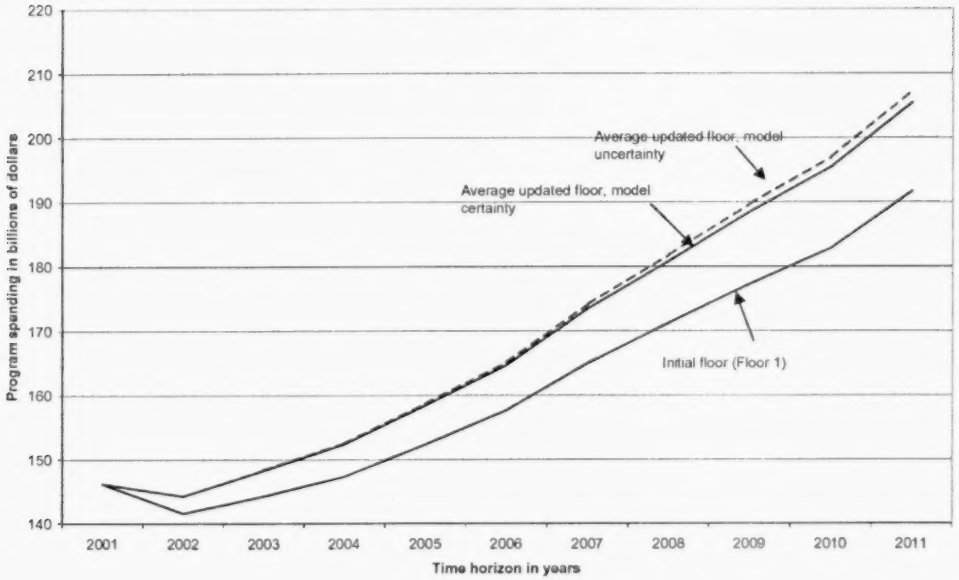
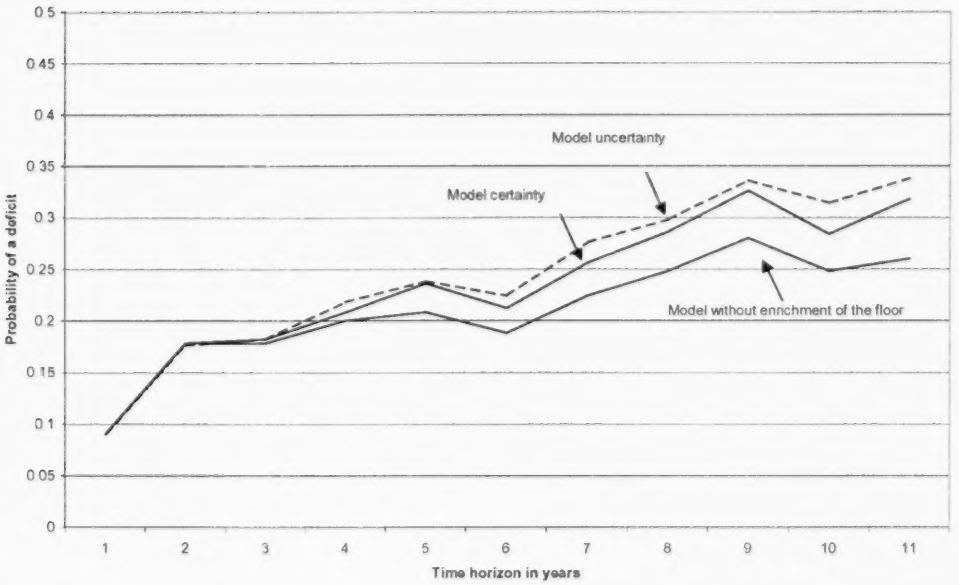


Figure 15. Probability of a deficit: New calibration



3. Conclusion

This study has shown that multi-period permanent commitments to tax and spending measures can be enhanced significantly without substantially raising the risk of a deficit outcome. This result is obtained in a framework where the fiscal authority is able to monitor the origin of the economic shocks and has model consistent expectations.

Secondly, this paper has evaluated whether knowledge about the persistence of the economic shocks is a key element driving this conclusion. To do this, we added to the economic uncertainty, uncertainty about the "true" persistence of the shocks and gauged whether, in that context, our strategy could still enhance the fiscal authority's pre-commitment to permanent measures without substantially increasing the risk of a deficit outcome.

We conclude that, as time goes on, the fiscal authority may find that it has been too optimistic in enhancing its multi-year commitment, which increases the probability of a deficit outcome. This illustrates the difficulties of making decisions under uncertainty and the potential cost of making mistakes. This also suggests the importance of prudent behavior when committing to long-term fiscal initiatives.

Our conclusion, however, depends on the way total uncertainty is controlled, and on other factors such as the calibration of our model and the ability of learning of the fiscal authority. Model uncertainty may well have less effect at medium-term horizons when these considerations are introduced.

Our framework of analysis leads naturally to one straightforward extension. In our study, there is no long-run (or average) tendency for the fiscal authority toward over- or under-prediction of the deviation of the growth rate from its steady state. We could extend our analysis by considering the impact of introducing a bias that led to systematic forecast errors *on average*. For example, the fiscal authority could be overly optimistic after both positive and negative shocks by overestimating the persistence of a positive shock, while underestimating the persistence of a negative shock. We leave this as an issue for future research.

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Appendixes

A1 Model uncertainty and forecast errors

The standard aggregate demand/supply framework of the model is conveniently described with the four following equations:

(A1) $$y_t = y_t^p + y_t^c$$

(A2) $$y_t^c = \alpha_1 y_{t-1}^c + \alpha_2 y_{t-2}^c + Z_t + \varepsilon_t^c$$

(A3) $$y_t^p = y_{t-1}^p + 0.25\mu_t + \varepsilon_t^p$$

(A4) $$\mu_t = \mu_t^{tar} + (1-\lambda)(\mu_{t-1} - \mu_{t-1}^{tar}) + \varepsilon_t^{\mu}.$$

These four equations have been described in Subsection 1.1. In the following, we illustrate how to model a situation where the fiscal authority makes forecast error in the sense that its expectation of the persistence of a shock is different from the model-consistent expectation.

Setting $\mu_\tau - \mu_\tau^{tar} = X_\tau$, the deviation of the trend from its deterministic component, into equation (A4), obtains:

(A5) $$X_t = (1-\lambda)X_{t-1} + \varepsilon_t^{\mu},$$

where (1-λ) is the persistence (autoregressive) coefficient of the deviations of the trend from its deterministic component. If λ = 1, the deviations would be purely white noise (the shocks have no persistence from one period to the next). If λ = 0, the deviation of the trend from its deterministic component would follow a random walk (the shocks are permanent).

In Section 1, we refer to (1-λ) as the "speed of reversion" of the shock of the growth rate of the potential output to its steady-state rate (μ^{tar}). As was said in Section 1, the fiscal authority can make mistakes about the "true" reversion speed. This mistake reflects a divergence between the model-consistent expectation of the deviation of the trend from its steady-state, and the fiscal authority expectation of this deviation.

The model-consistent expectation of the deviation of the trend from its deterministic component, conditional on previous period observation $[X_{t-1}]$, is obtained by taking the conditional expectation of both sides of equation (A5):

(A6) $$E_{t-1}[X_t] = (1-\lambda)X_{t-1}.$$

(Note in passing that the *unconditional* expectation of the process given in (A5) is $E[X_t] = 0$ such that $\mu_t = \mu_t^{tar}$; without *a priori* knowledge of the deviation from the

deterministic trend, all we can say is that the stochastic trend will be on average equal to the deterministic trend.)

According to equation (A6), a share (1-λ) of the previous period deviation of the trend from the deterministic component is expected to persist in this period, which is "consistent" with the statement in equation (A5). Figure A1 illustrates the model-consistent expectations based on different values of the persistence coefficient, (1-λ). For the same initial deviation (distance between points B and A) of the trend μ, from its deterministic component μ^{tar}, a lower value of the parameter λ implies a greater expected persistency, over time, of this deviation. (For convenience, it has been assumed in the graph that μ^{tar} is constant over the planning horizon. As said in Subsection 1.1, in our stochastic model, μ^{tar} slightly decreases over time.)

Equation (A6) gives the model-consistent expectation of the deviation of the growth rate from its deterministic trend. The fiscal authority, on the other hand, is assumed to form its expectation according to:

(A7) $$E_{t-1}^{fisc}[X_t] = (1-\lambda^{fisc})X_{t-1}.$$

Comparing equations (A6) and (A7), the fiscal authority's conditional expectation coincides with the model-consistent expected value if $\lambda^{fisc} = \lambda$. This is the assumption that is done under model certainty. Discrepancies between λ^{fisc} and the "true" λ are symptomatic of model uncertainty. In other words, the fiscal authority does not know the true value of the persistence coefficient, but evaluates it as $(1-\lambda^{fisc})$.

Based on equations (A6) and (A7), the forecast error of the fiscal authority due to model uncertainty and conditional on the previous-period deviation from the deterministic trend is given by the expression:

(A8) Conditional forecast error = $$E_{t-1}^{fisc}[X_t] - E_{t-1}[X_t] = (\lambda - \lambda^{fisc})X_{t-1}.$$

Figure A2 illustrates these forecast errors. The fiscal authority is too optimistic (positive-sign forecast error, or over-prediction) after the arrival of positive shocks (point A) and too pessimistic (negative-sign forecast error, or under-prediction) after the arrival of negative shocks (point B). This means that periods of slower growth than expected (point B) will be followed by an over-pessimistic projection for the growth rate of potential output, whereas periods of higher growth rate than expected (point A) will be followed by an over-optimistic projection for the growth rate. These under/over projections will have consequences for the fiscal planning. Periods of higher growth than expected will be followed by a budget planning that overestimates the projections for tax revenues (TAX) (and underestimates the projections for the automatic stabilization spending (PSC)). Periods of lower growth than expected will have the opposite effect.

Figure A1. Model-consistent expectations of deviations from trend

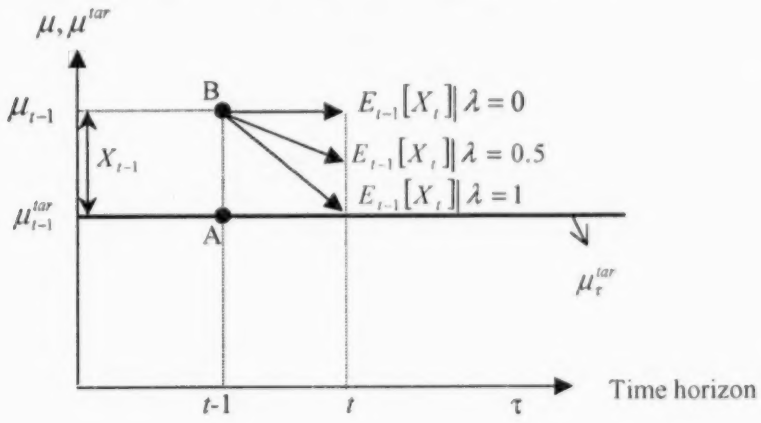
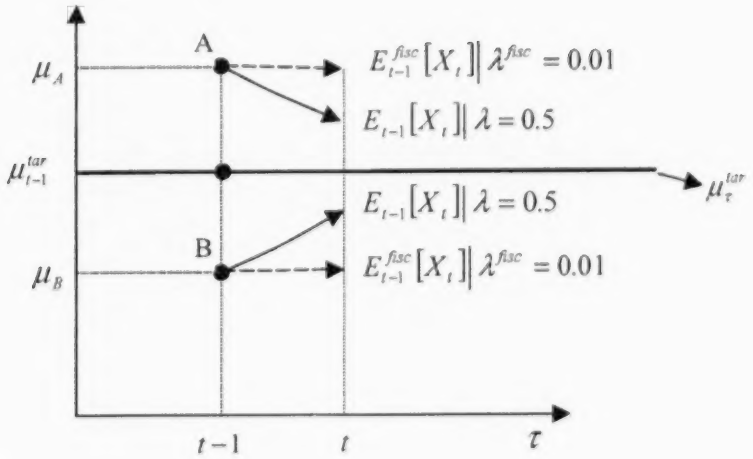


Figure A2. Conditional forecast errors



Two implications derive from this. First, the fiscal authority does not make systematic forecast errors on the trend but overestimates the *persistence* of the shocks on the trend. This is so because positive and negative shocks arrive with the same probability. Hence, there is no long-run (or average) tendency for the fiscal authority toward the over- or under-prediction of the deviation from the trend or over- or under-estimation of the projections for tax and program spending. In other words, the fiscal authority unconditional forecast error due to model uncertainty is zero; in average over the long run, $E^{fisc}[X_t] = E[X_t] = 0$ or $E^{fisc}(\mu) = E(\mu) = \mu^{tar}$.

The second implication is obvious from the observation of equations (A5) and (A8). The variance of ε^{μ}, through its influence on the variance of the deviations X from the deterministic trend, has a direct impact on the magnitude of the conditional forecast errors due to model uncertainty. In particular, an increase in the variance of ε^{μ} amplifies the fluctuations in X, and therefore magnifies the conditional forecast errors of the fiscal authority. Hence, we should expect an amplification of the fiscal planning consequences of model uncertainty with a larger variance of ε^{μ}. This leads us, in Appendix A2 to the issue of the variance that one should impose on ε^{μ}.

A.2 The calibration of the "output" shocks

From equation (A1), the variance of the growth rate of the real output is given by:

(A9) $$Var(dy) = Var(dy^{p}) + Var(dy^{c}), \qquad [cov(dy^{p}, dy^{c}) = 0]$$

where dy is the (annualized) effective rate of growth of real GDP between any two consecutive quarters [*i.e.*, $dy = 4(y_t - y_{t-1})$ – where "y_t" is the log of the real GDP in quarter τ] , and dy^{p} and dy^{c} are the annualized rates of growth of potential and cyclical outputs. Our model is calibrated such that the variance of the growth rate of real output (approximately) matches its historical value [$Var(dy) = 3.75^2$ over the period 1961Q1 to 1999Q4].

The decomposition of the historical variance of the real output into a variance for the growth rate of the potential and cyclical outputs is intricately linked to the statistical methodology used to decompose the real output into potential and cyclical outputs (*e.g.*, the Hodrick-Prescott filter or the Unobservable Component method). At any rate, this statistical decomposition is imprecise given that, by definition, the potential and cyclical outputs are unobservable.

Using the Unobservable Component methodology, we obtain a "historical" decomposition between potential and cyclical output that entails a variance for the growth rate of the potential output of about 3.3^2 and a variance for the rate of growth of the cyclical output of about 1.85^2. Our stochastic simulation model has been calibrated to approximate these "historical" variances using a matching-moment procedure.

For example, a simple examination of equations (A3) and (A4), shows that $Var[dy^p]$ must be related to the variance of ε^μ and ε^p, and thus, the calibration procedure involves finding the variances of ε^μ and ε^p (*i.e.*, σ_μ^2 and σ_p^2) that generate, over a large number of simulations of our model, an empirical variance for dy^p that matches its "historical" value of 3.3^2. As shown in Appendix A3 the statistical relation between σ_μ^2, σ_p^2 and $Var[dy^p]$ is:

$$Var(dy^p) = \frac{\sigma_\mu^2}{(1-(1-\lambda)^2)} + 16\sigma_p^2, \qquad \text{(A10)}$$

and so, there is an infinite number of combinations of $(\sigma_\mu;\sigma_p)$ that may satisfy this relationship.

Fortunately, the unobserved component methodology not only decomposes the real output into series for y^p and y^c, but it also generates a series on μ, from which we can compute an "historical" variance given by $Var(\mu) = (0.866)^2 = 0.75$ over the period 1961Q1-1999Q4. This permits us to pin down one specific combination of $(\sigma_\mu;\sigma_p)$ that may satisfy equation (A10). Indeed, as shown in Appendix A3, the first term on the r.h.s. of equation (A10) is equal to $Var(\mu)$, and thus, for a given λ (=0.5), this permits us to calibrate (*i.e.*, to "fix" the value of) σ_μ^2 such that our model will generate an empirical variance for μ matching its historical value. This entails setting the variance for ε^μ at $\sigma_\mu^2 = (0.75)^2$.

According to equation (A10), we can now impose $\sigma_p^2 = (0.7960)^2$, which, together with $\sigma_\mu^2 = (0.75)^2$ will generate an empirical variance for dy^p that will match its historical value of 3.3^2. This therefore provides us with one specific calibration for the pair $(\sigma_\mu;\sigma_p)$ that can match the "historical" variance of dy^p.

A.3 Decomposition of real GDP growth

From equation (A1), the (annualized) effective rate of growth of the real GDP between quarters *t* and *t-1* can be decomposed into the growth of the potential and cyclical outputs as:

$$dy_t = dy_t^c + dy_t^p \qquad \text{(A11)}$$

where $dy = 4(y_t - y_{t-1})$ and "y" is the log of the real GDP.

We can show that the growth of the potential output can itself be decomposed into three components, a time-varying deterministic drift, μ_t^{var}, the deviation, X, of the (stochastic) trend, μ, to the deterministic drift, and a white noise. We show this by rewriting equation (A3) as follows:

(A12) $$dy_t^p = 4(y_t^p - y_{t-1}^p) = \mu_t + 4\varepsilon_t^p$$

where dy_t^p is the (annualized) rate of growth of potential output between quarter (t) and quarter (t-1).

Substituting μ_t given in equation (A4) into (A12), obtains:

(A13) $$dy_t^p = \mu_t^{tar} + (1-\lambda)(\mu_{t-1} - \mu_{t-1}^{tar}) + \varepsilon_t^{\mu} + 4\varepsilon_t^p$$

which, given (A5), can be rewritten as:

(A14) $$dy_t^p = \mu_t^{tar} + X_t + 4\varepsilon_t^p.$$

Hence, as said above, the rate of growth of potential output depends on three components:

1.The time-varying deterministic drift, μ_t^{tar},

2. The deviation, X_t, of the (stochastic) trend, μ_t, from its deterministic component μ_t^{tar}, whose process is given by: $X_t = (1-\lambda)X_{t-1} + \varepsilon_t^{\mu}$ [equation (A5)],

3. The white noise on the level of the potential output, ε_t^p.

Based on (A12), the variance of dy^p is given as:

(A15) $$Var(dy^p) = Var(\mu + 4\varepsilon^p) = Var(\mu) + 16(\sigma_p^2),$$

given that $Cov(\mu, \varepsilon^p) = 0$.

Or, alternatively, based on (A14):

(A15bis) $$Var(dy^p) = Var(X) + 16(\sigma_p^2) \quad \text{(given that } Cov(X, \varepsilon^p) = 0).$$

Comparing (A15) with (A15bis) implies that Var(X) = Var(μ).

Var(X) is the variance of the AR(1) process given in equation (A6), and obtained by the formula (given in any time-series textbook):

(A16) $$Var(X) = Var(\mu) = \frac{Var(\varepsilon^{\mu})}{(1-(1-\lambda)^2)} = \frac{\sigma_{\mu}^2}{(1-(1-\lambda)^2)} \quad for\ \lambda \neq 0.$$

For $\lambda=0$, the process in equation (A16) is a random walk, and its variance is a function of time, t:

$$Var(X_t) = Var(\mu_t) = \sigma_\mu^2 t$$

Substituting equation (A16) into (A15bis), the variance for dy^P may therefore be rewritten (for $\lambda \neq 0$) as:

$$Var(dy^P) = \frac{(\sigma_\mu^2)}{(1-(1-\lambda)^2)} + 16(\sigma_p^2)$$

There is an infinite number of combinations of $(\sigma_\mu^2; \sigma_p^2)$ that may give a specific $Var(dy^P)$ for a given λ.